Convolutional Networks

Lecture slides for Chapter 9 of *Deep Learning*

Ian Goodfellow

2016-09-12

Convolutional Networks

- Scale up neural networks to process very large images / video sequences
    - Sparse connections
    - Parameter sharing
- Automatically generalize across spatial translations of inputs
- Applicable to any input that is laid out on a grid (1-D, 2-D, 3-D, ...)

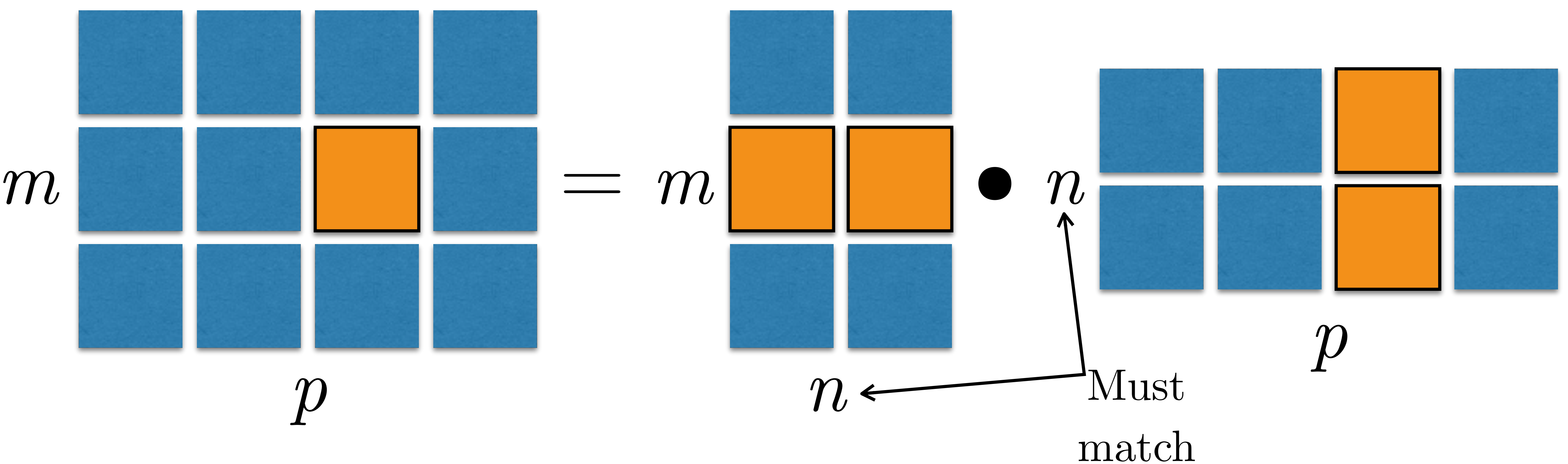
Key Idea

- Replace matrix multiplication in neural nets with convolution
- Everything else stays the same
  - Maximum likelihood
  - Back-propagation
  - etc.

Matrix (Dot) Product

$$\boldsymbol{C} = \boldsymbol{A}\boldsymbol{B}. \tag{2.4}$$

$$C_{i,j} = \sum_k A_{i,k} B_{k,j}. \tag{2.5}$$



(Goodfellow 2016)

Matrix Transpose

$$(\boldsymbol{A}^\top)_{i,j} = A_{j,i}. \tag{2.3}$$

$$\boldsymbol{A} = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \\ A_{3,1} & A_{3,2} \end{bmatrix} \Rightarrow \boldsymbol{A}^\top = \begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} \\ A_{1,2} & A_{2,2} & A_{3,2} \end{bmatrix}$$

Figure 2.1: The transpose of the matrix can be thought of as a mirror image across the main diagonal.

$$(\boldsymbol{A}\boldsymbol{B})^\top = \boldsymbol{B}^\top \boldsymbol{A}^\top. \tag{2.9}$$

(Goodfellow 2016)

2D Convolution

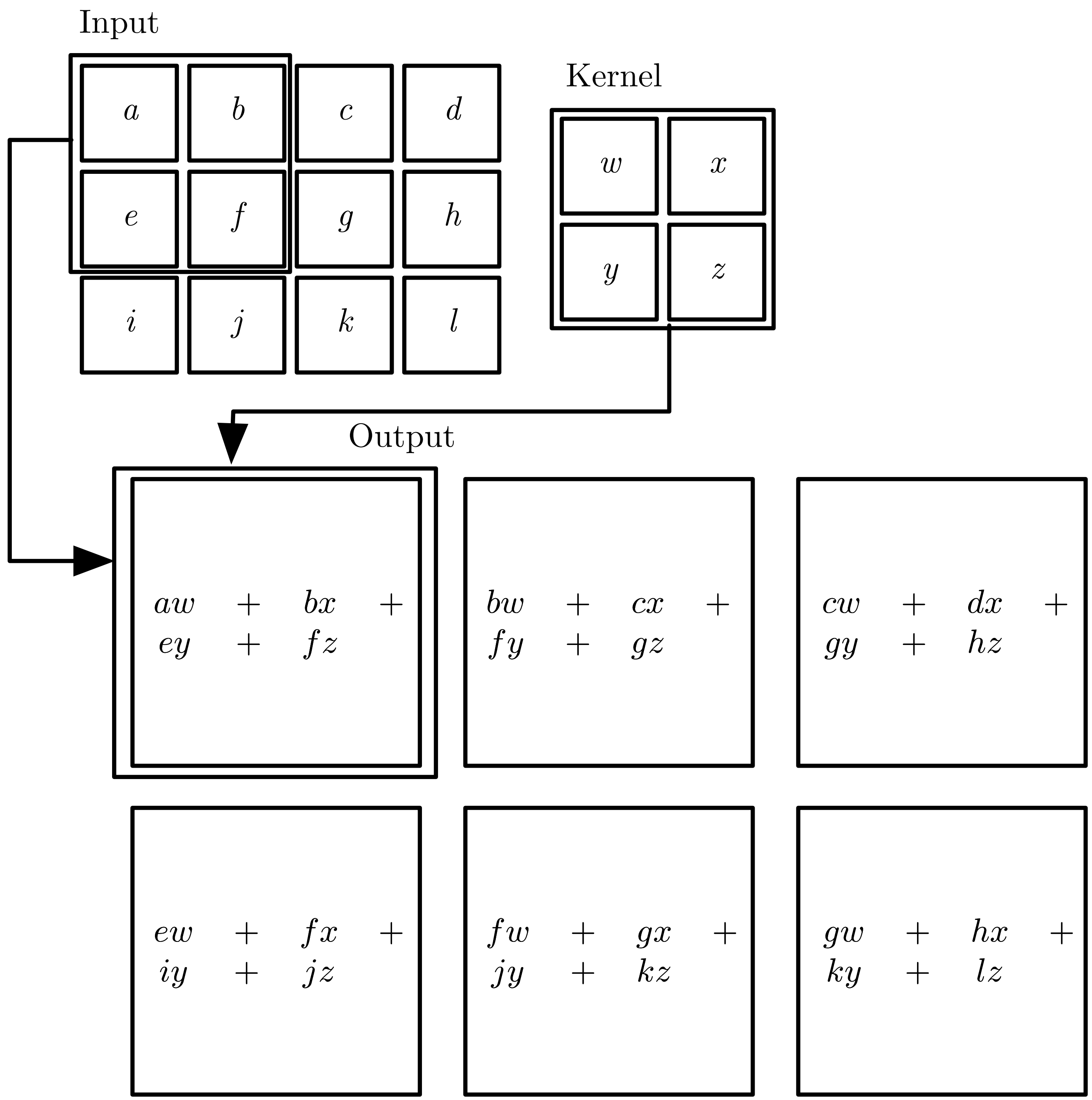


Figure 9.1

Three Operations

- Convolution: like matrix multiplication
  - Take an input, produce an output (hidden layer)
- "Deconvolution": like multiplication by transpose of a matrix
  - Used to back-propagate error from output to input
  - Reconstruction in autoencoder / RBM
- Weight gradient computation
  - Used to backpropagate error from output to weights
  - Accounts for the parameter sharing

Sparse Connectivity

Sparse connections due to small convolution kernel

Dense connections

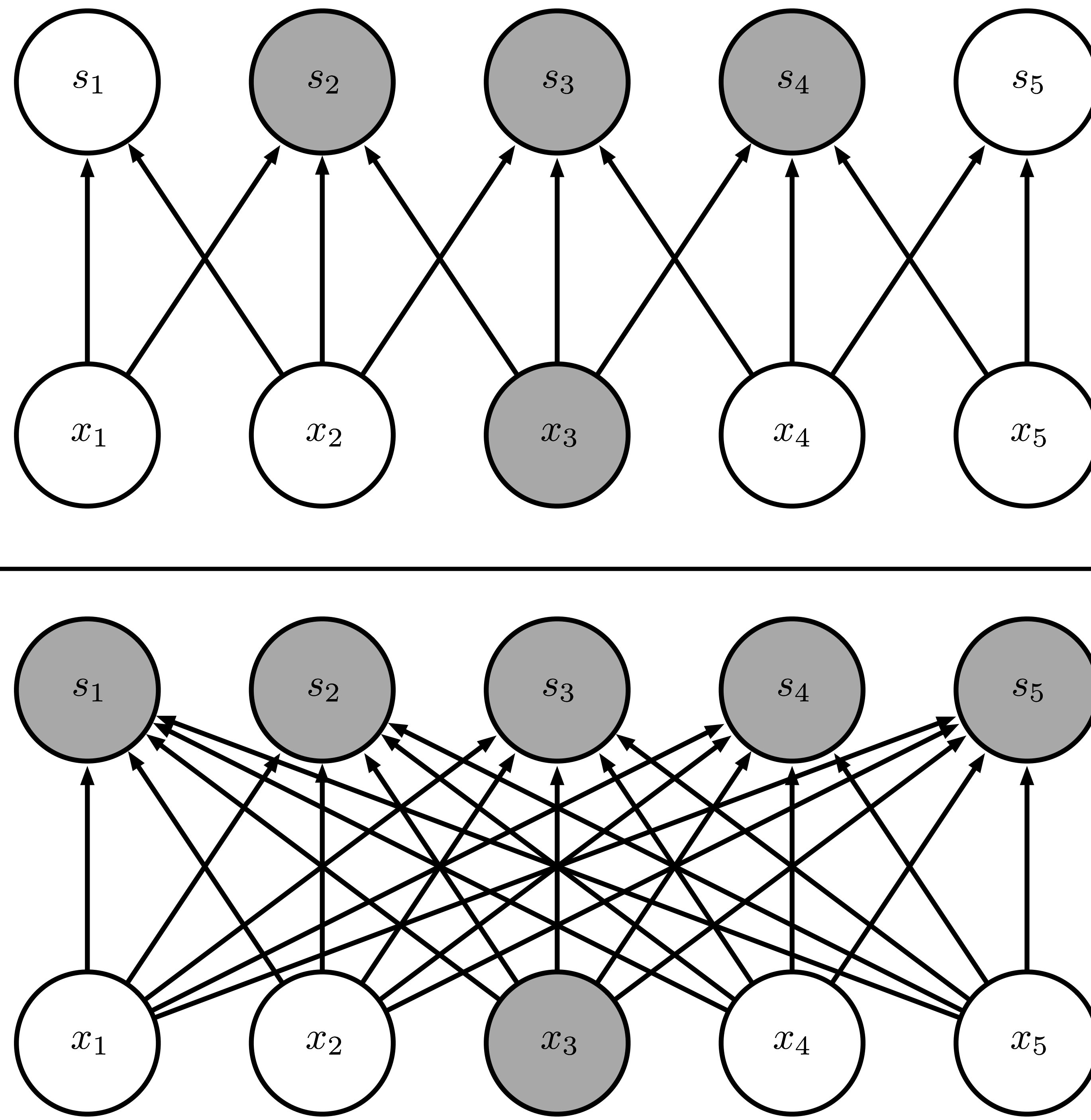


Figure 9.2

(Goodfellow 2016)

Sparse Connectivity

Sparse connections due to small convolution kernel

Dense connections

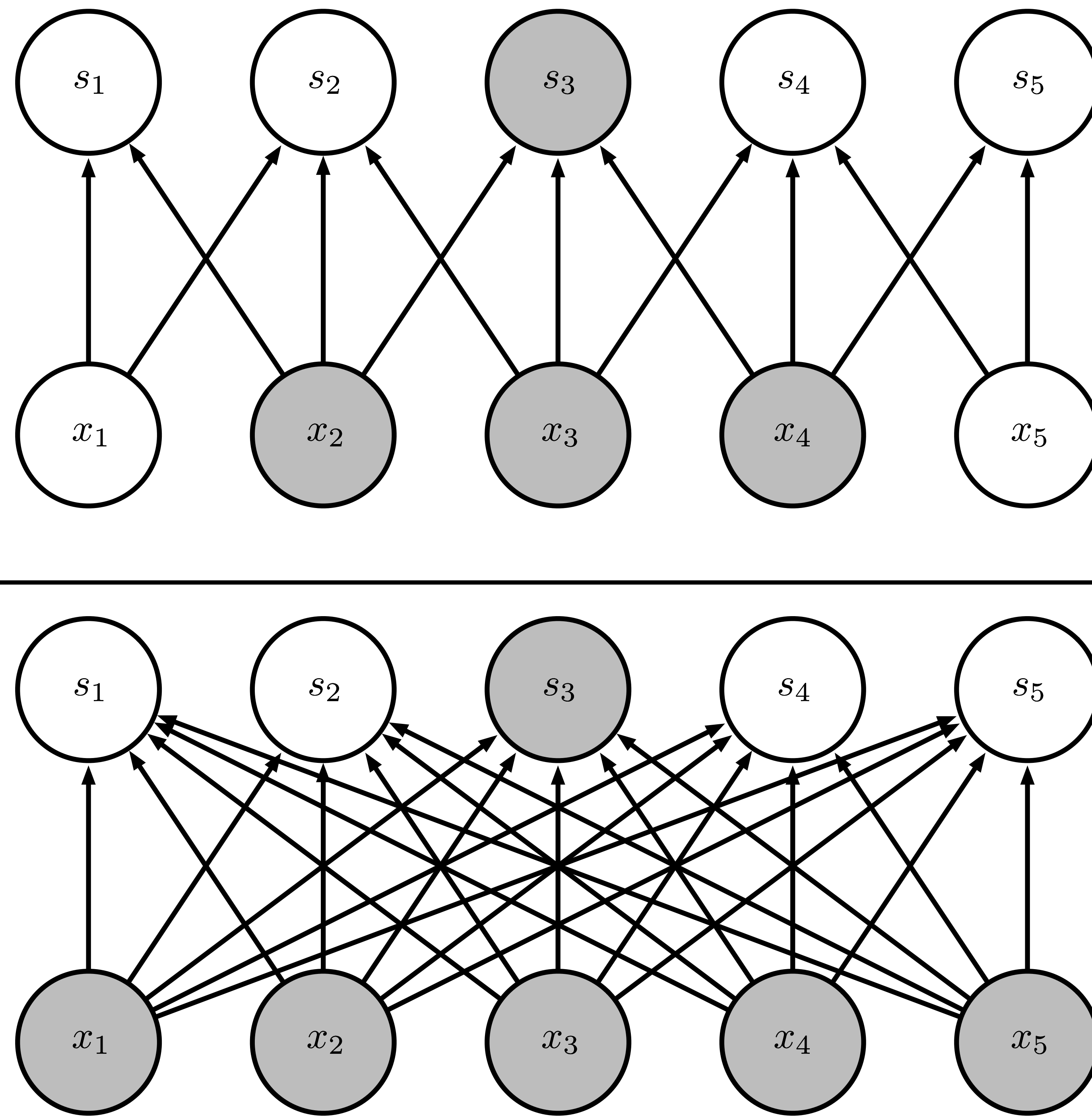


Figure 9.3

(Goodfellow 2016)

Growing Receptive Fields

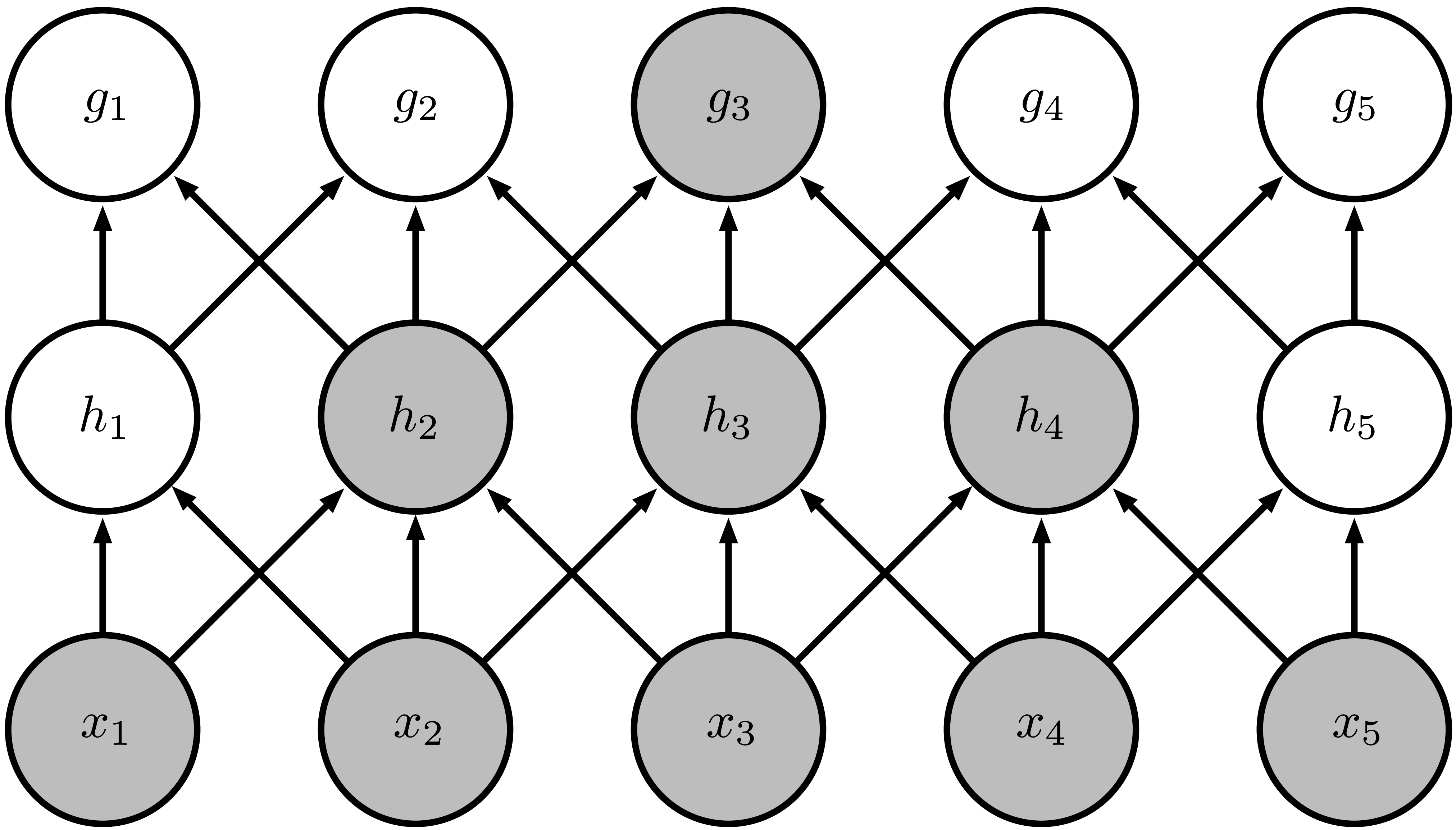


Figure 9.4

Parameter Sharing

Convolution shares the same parameters across all spatial locations

Traditional matrix multiplication does not share any parameters

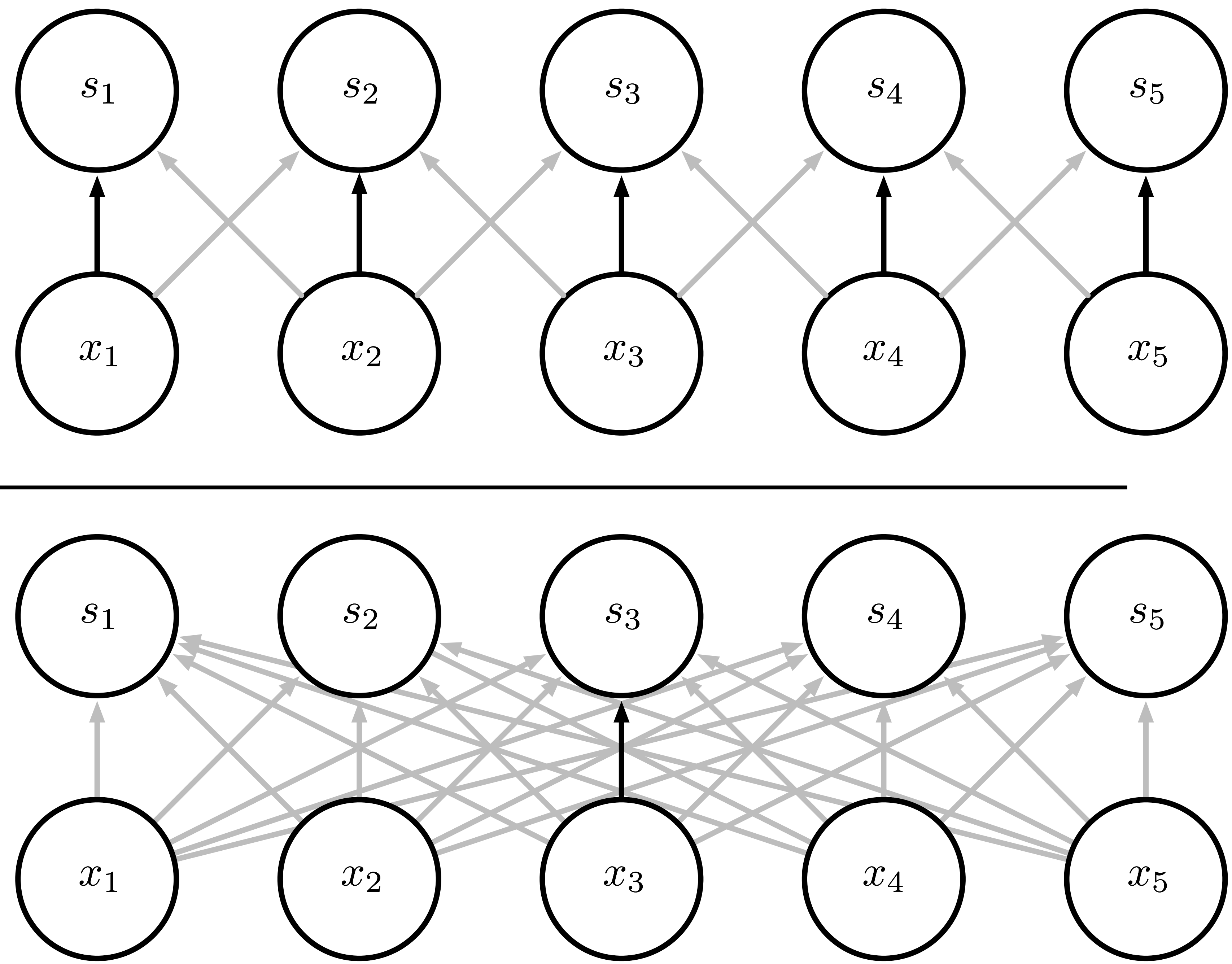


Figure 9.5

(Goodfellow 2016)

Edge Detection by Convolution

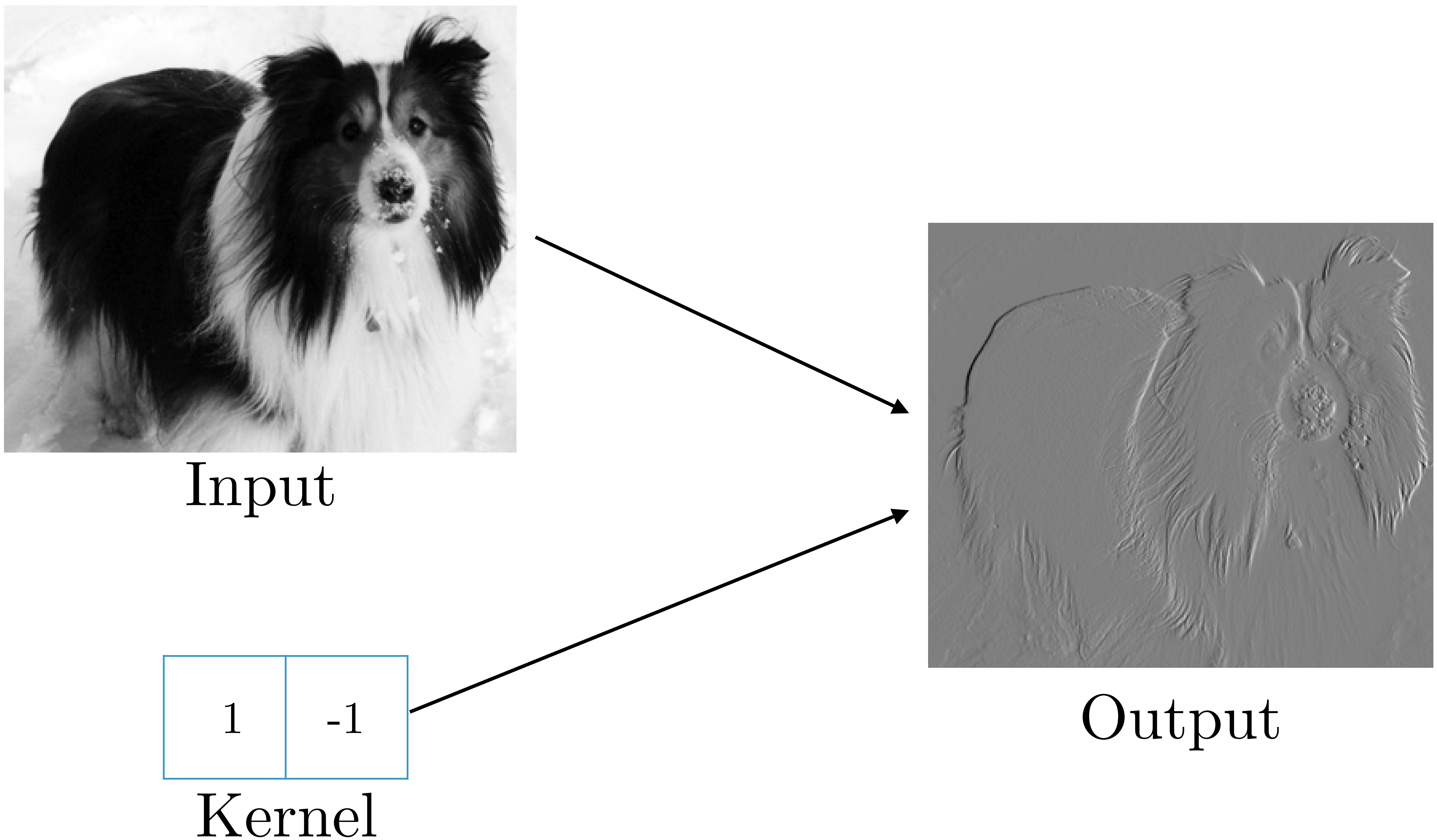


Figure 9.6

(Goodfellow 2016)

Efficiency of Convolution

Input size: 320 by 280

Kernel size: 2 by 1

Output size: 319 by 280

	Convolution	Dense matrix	Sparse matrix
Stored floats	2	319*280*320*280 $>$ 8e9	2*319*280 = 178,640
Float muls or adds	319*280*3 = 267,960	$>$ 16e9	Same as convolution (267,960)

(Goodfellow 2016)

Convolutional Network Components

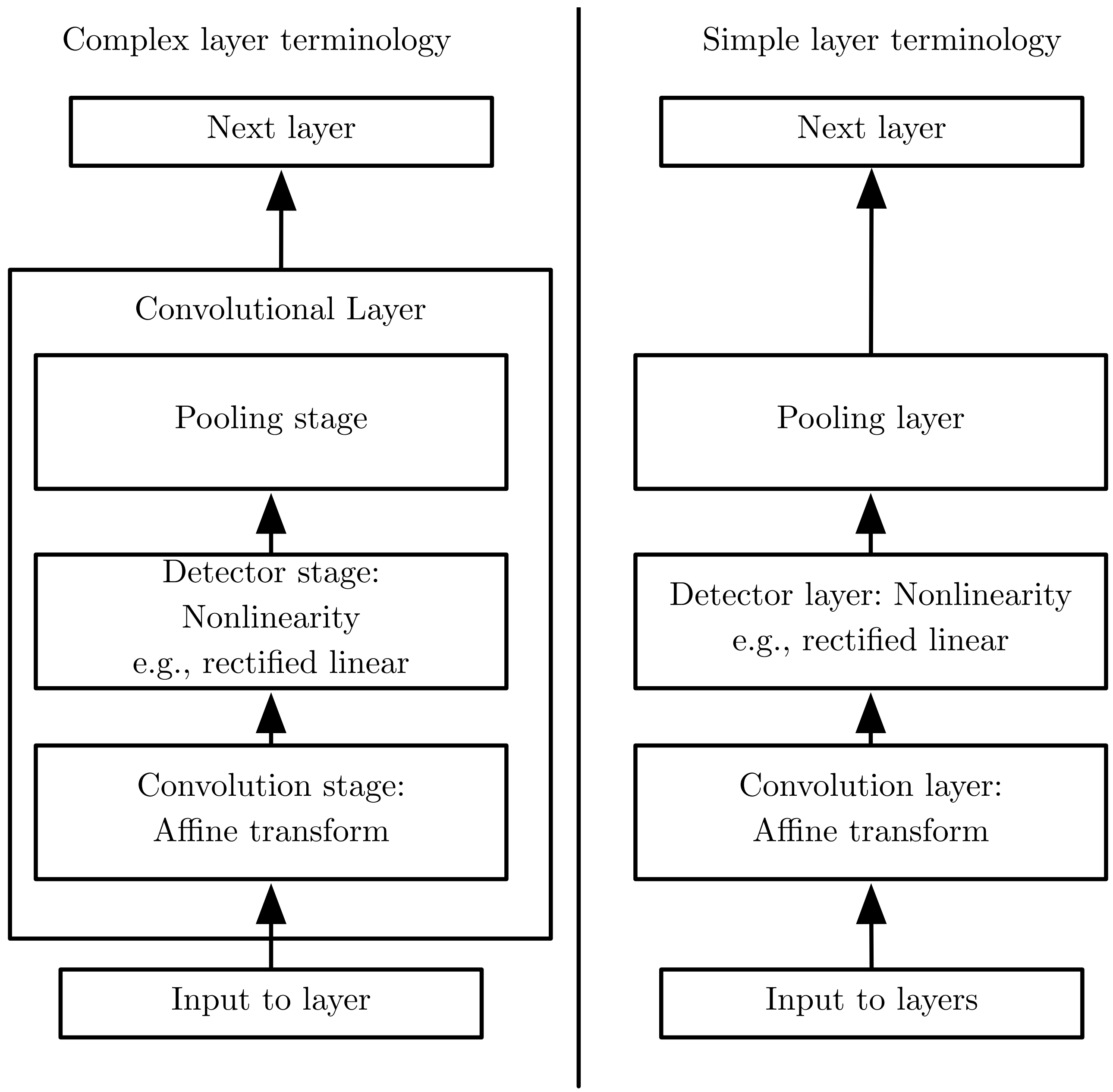


Figure 9.7

(Goodfellow 2016)

Max Pooling and Invariance to Translation

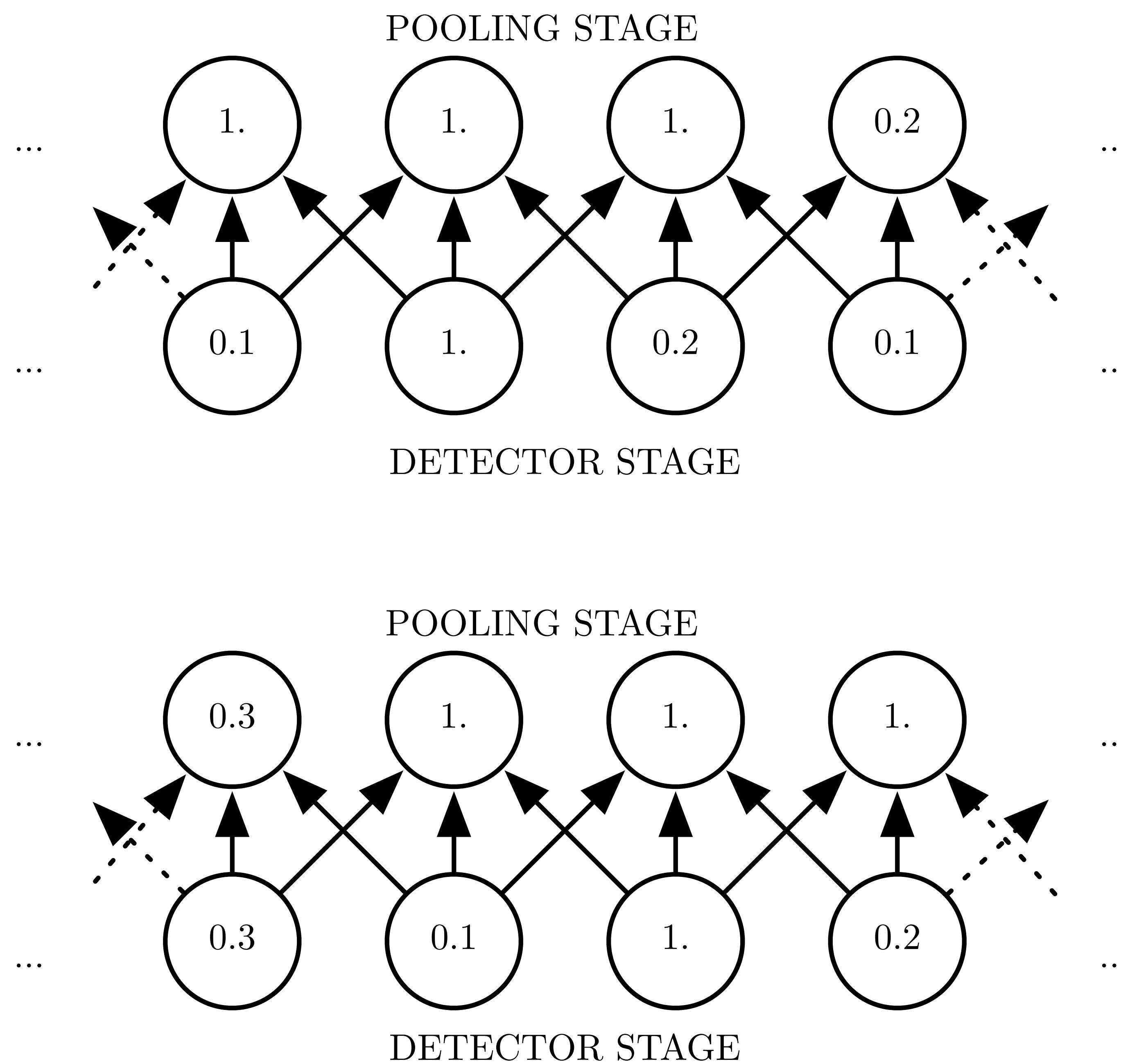


Figure 9.8

(Goodfellow 2016)

Cross-Channel Pooling and Invariance to Learned Transformations

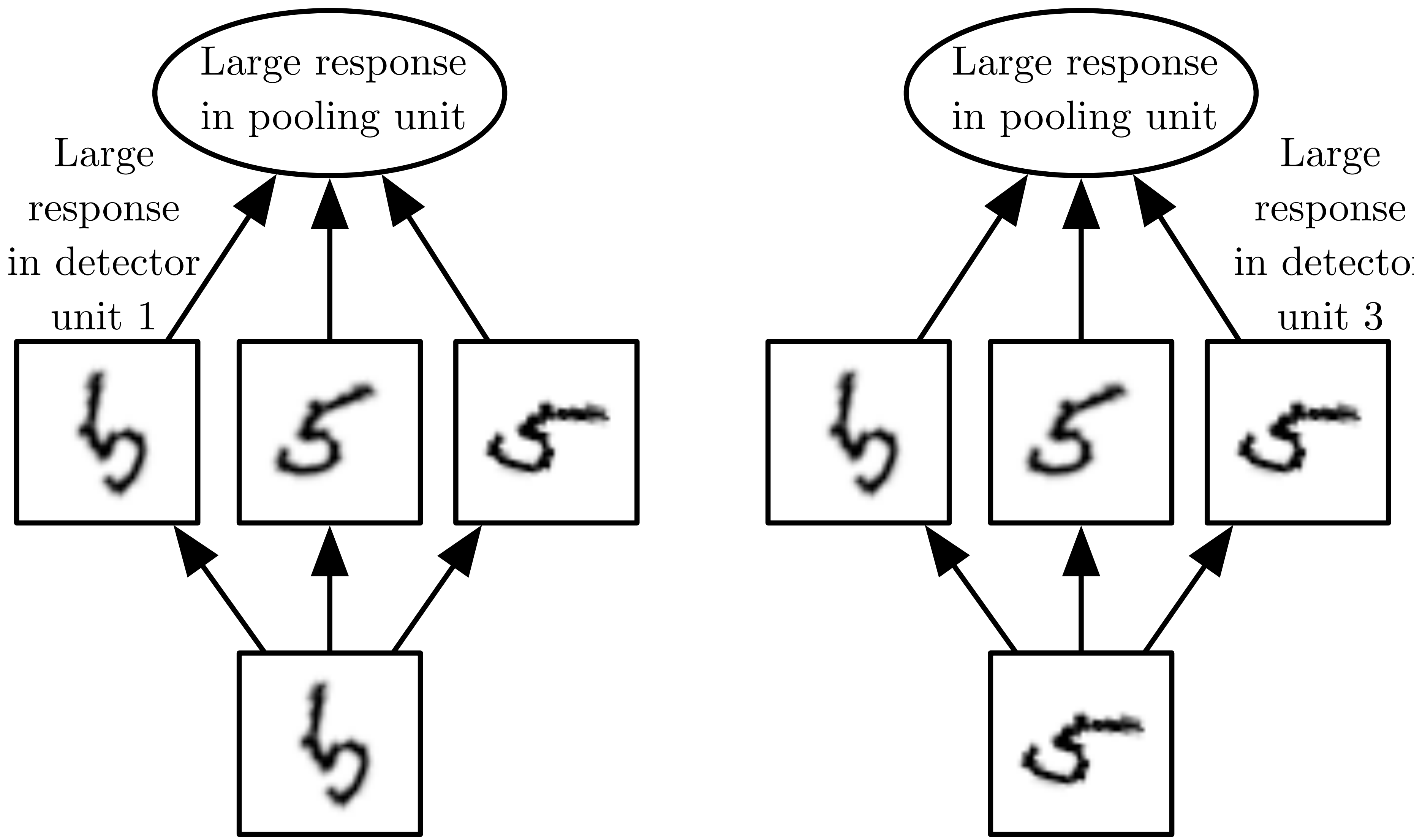


Figure 9.9

(Goodfellow 2016)

Pooling with Downsampling

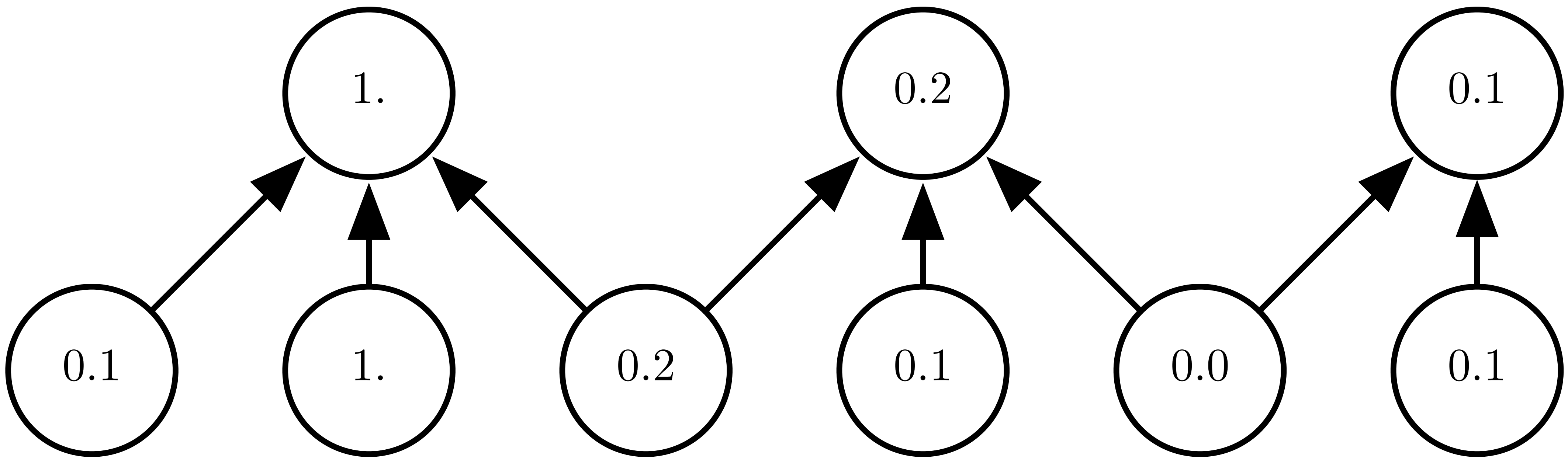


Figure 9.10

(Goodfellow 2016)

Example Classification Architectures

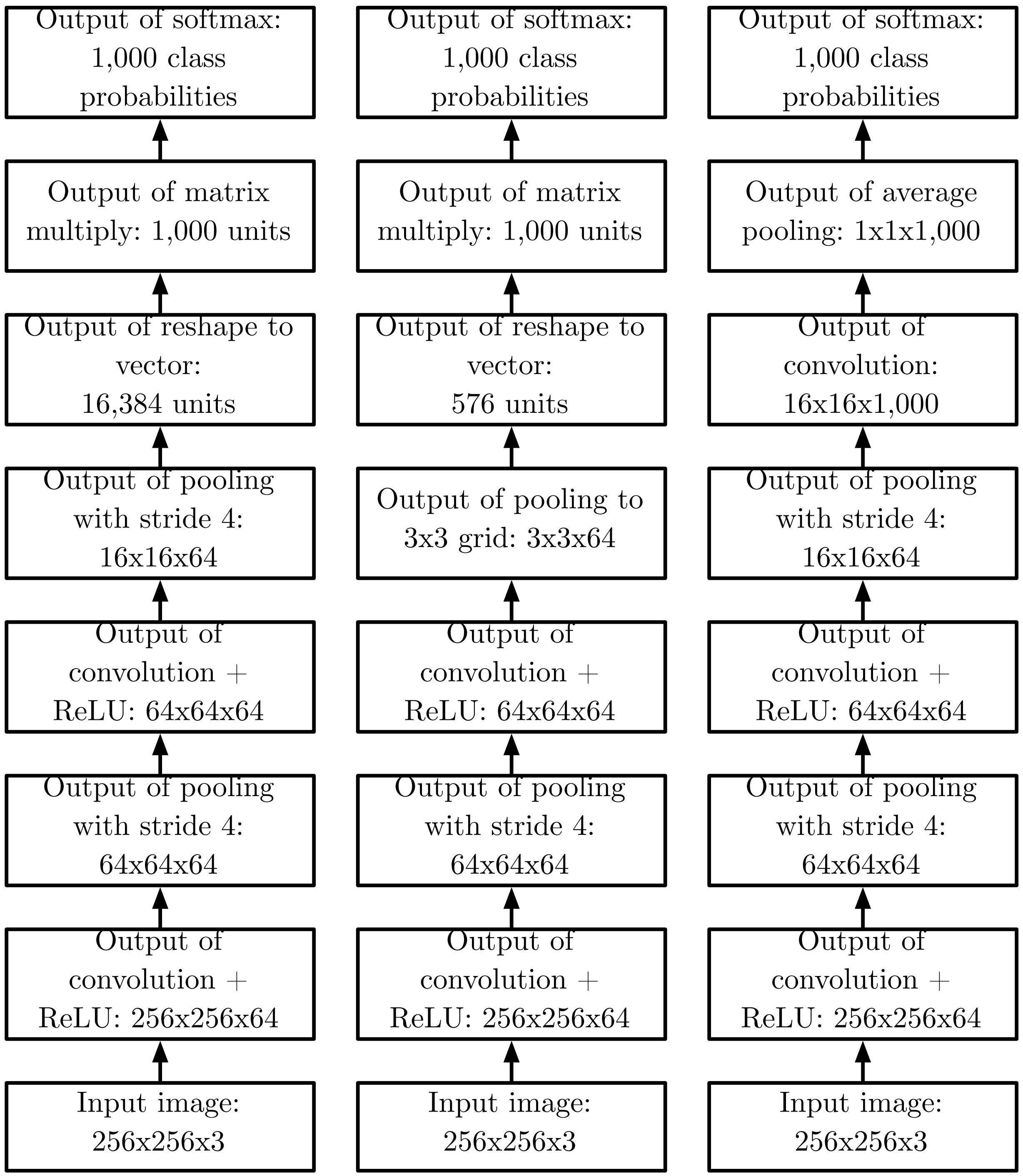


Figure 9.11

(Goodfellow 2016)

Convolution with Stride

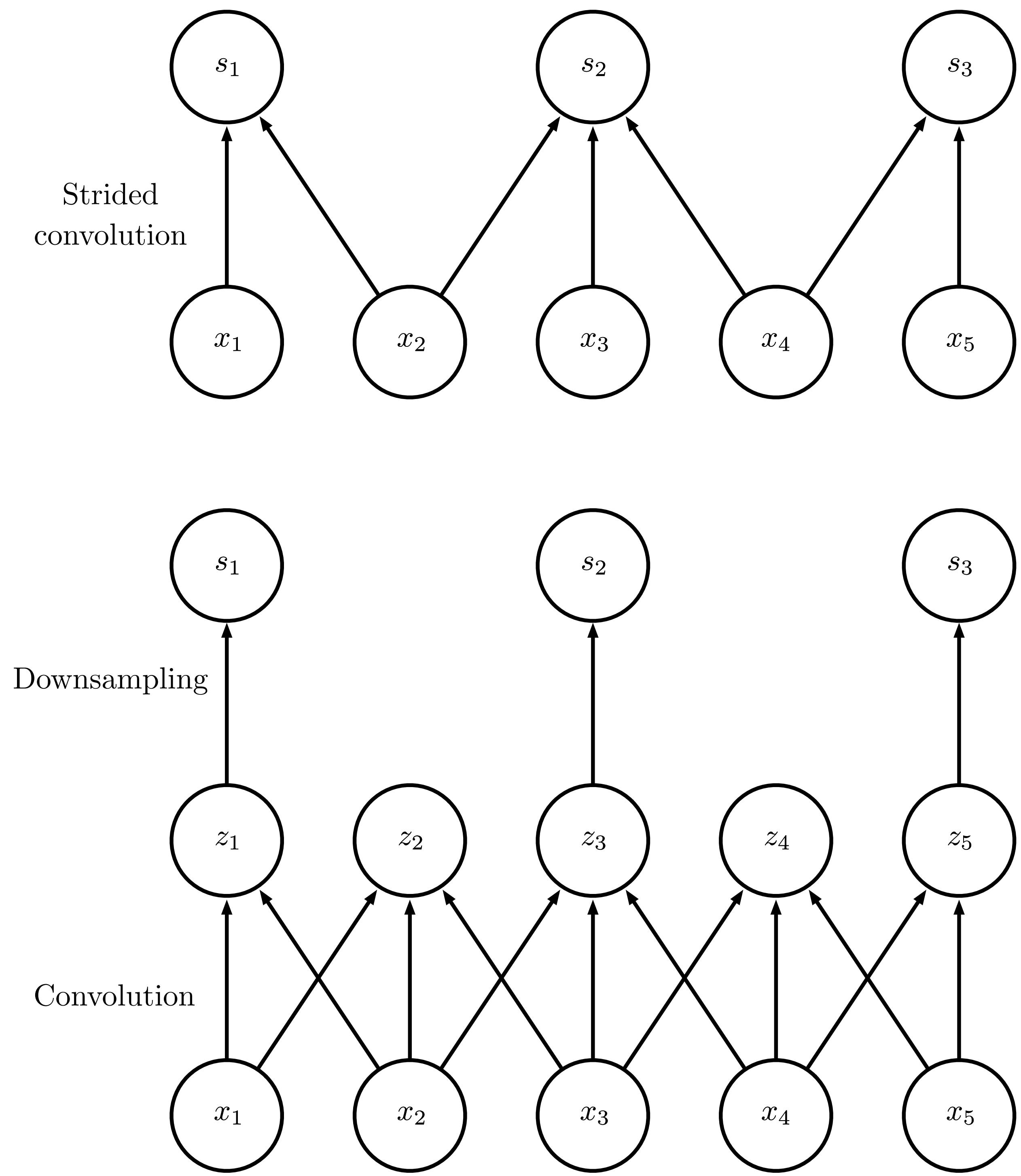


Figure 9.12

(Goodfellow 2016)

Zero Padding Controls Size

Without zero padding

With zero padding

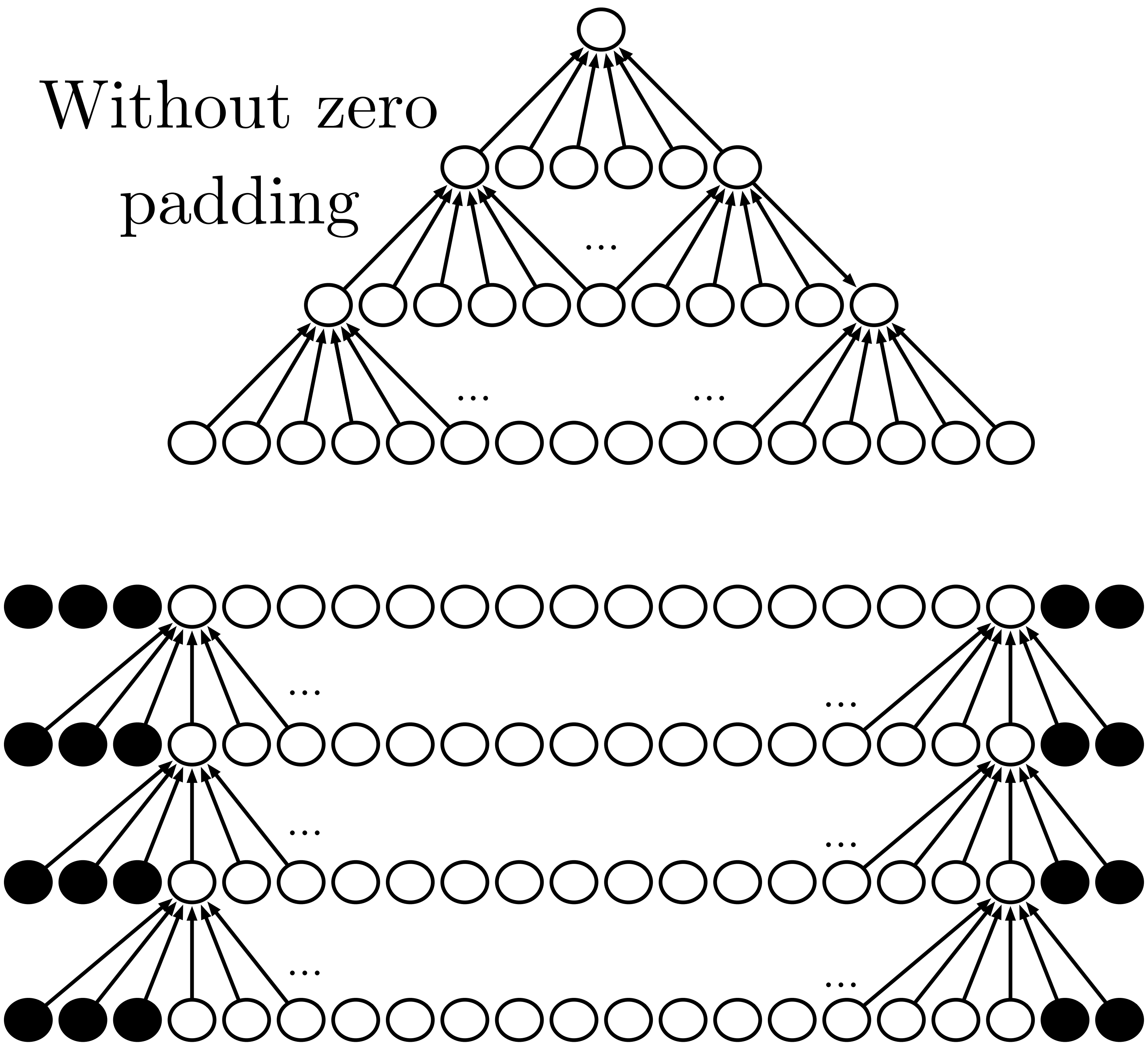
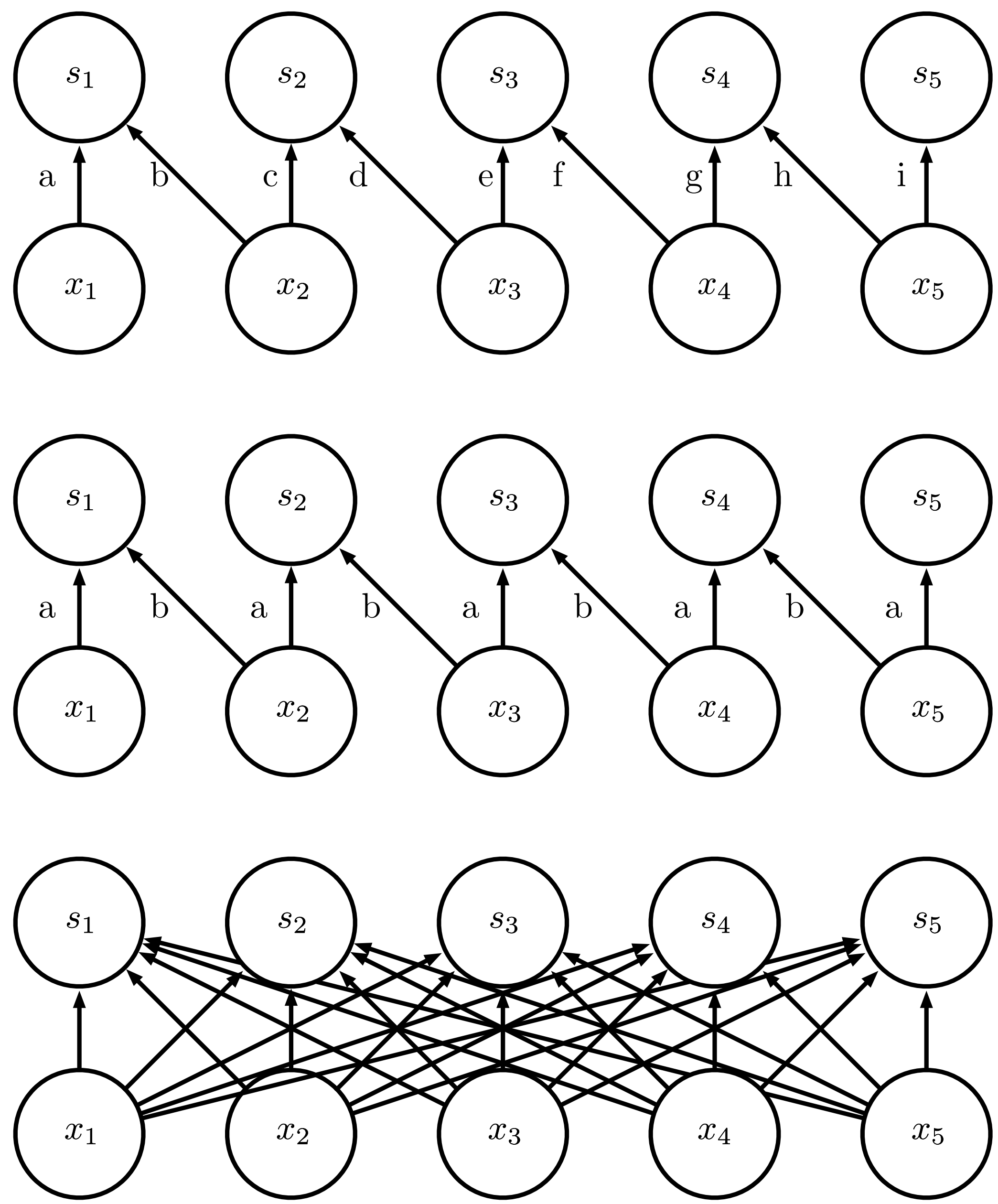


Figure 9.13

(Goodfellow 2016)

Kinds of Connectivity



Local connection: like convolution, but no sharing

Convolution

Fully connected

Figure 9.14

(Goodfellow 2016)

Partial Connectivity Between Channels

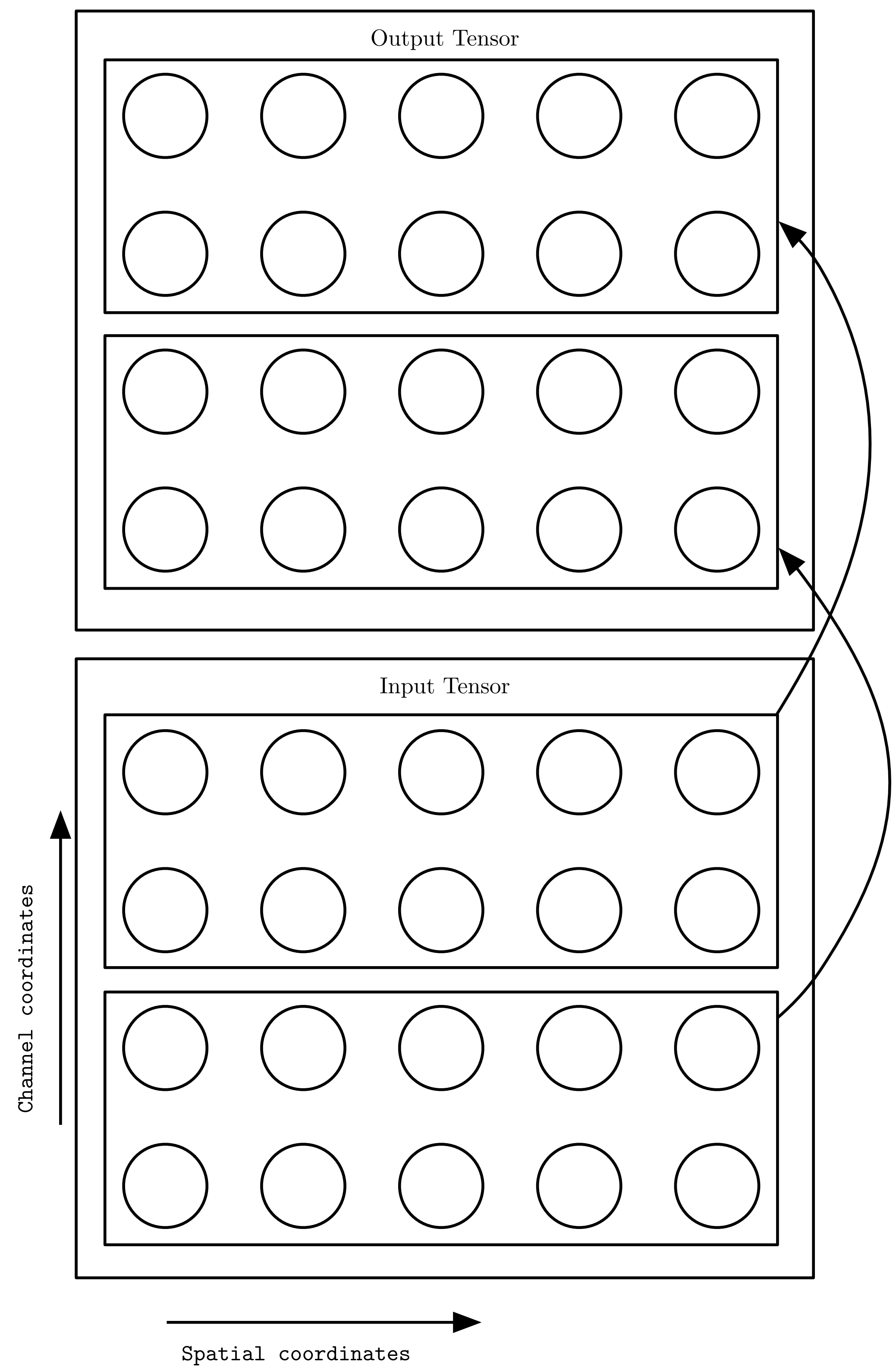
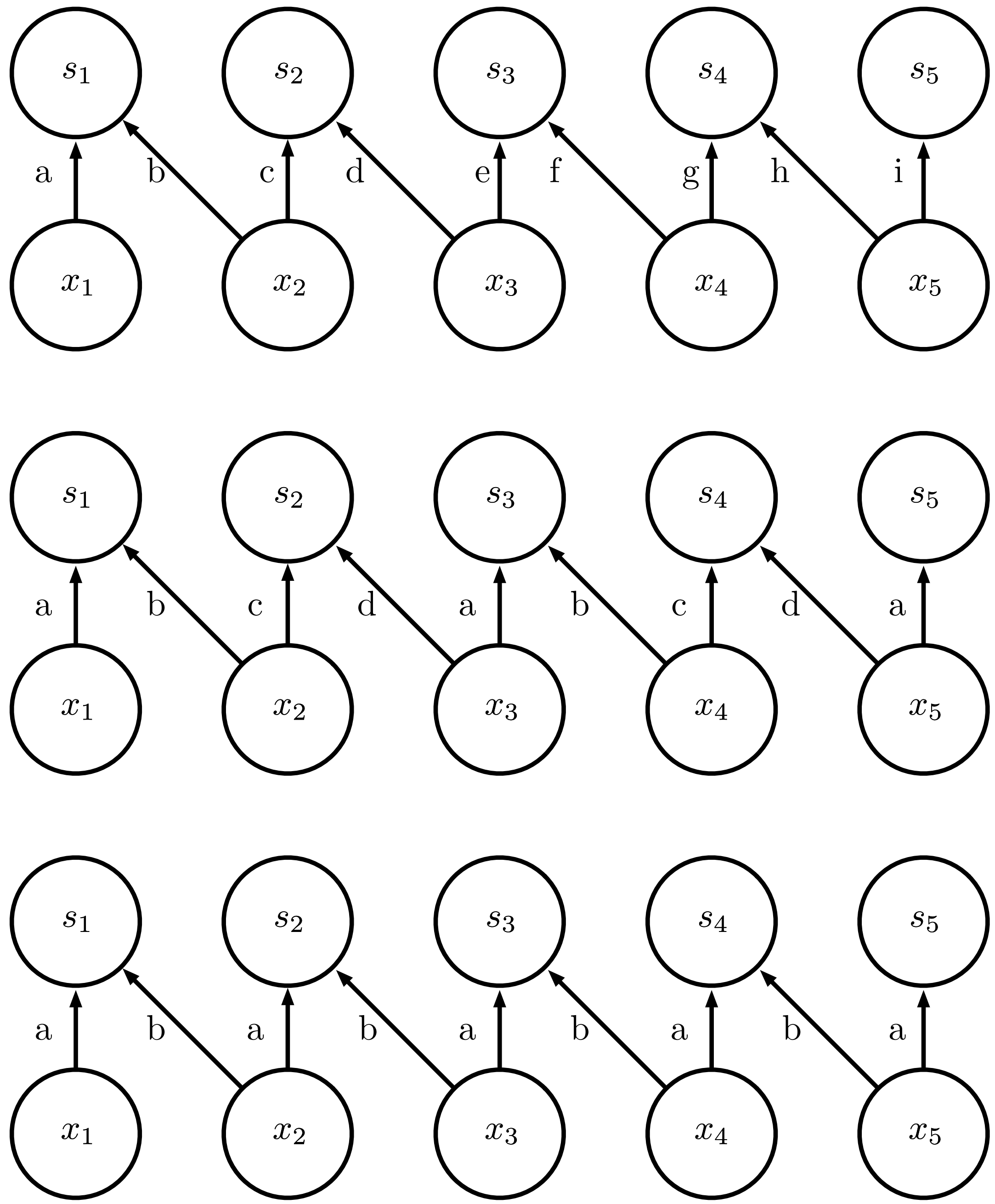


Figure 9.15

(Goodfellow 2016)

Tiled convolution



Local connection (no sharing)

Tiled convolution (cycle between groups of shared parameters)

Convolution (one group shared everywhere)

Figure 9.16

Recurrent Pixel Labeling

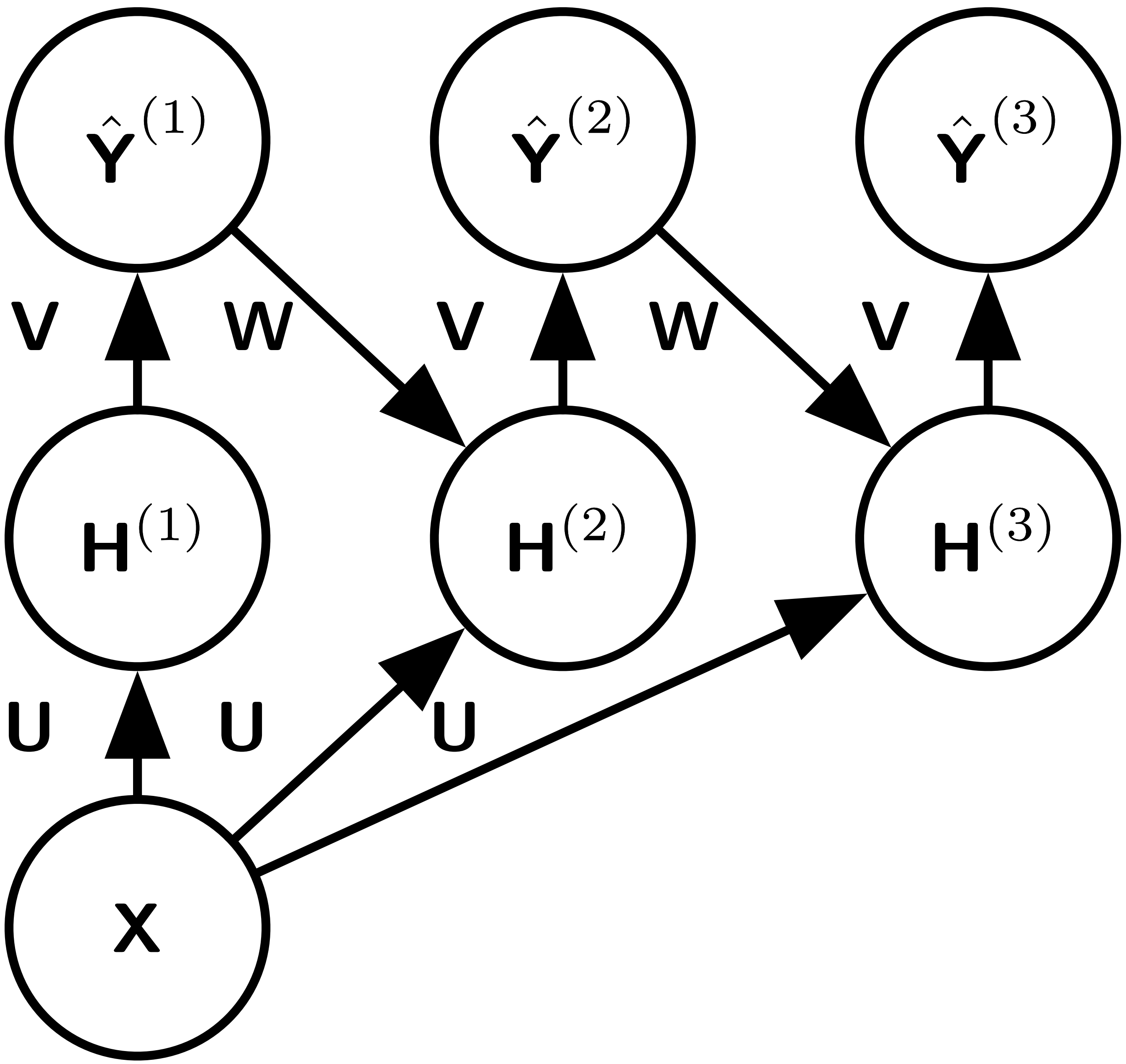


Figure 9.17

(Goodfellow 2016)

Gabor Functions

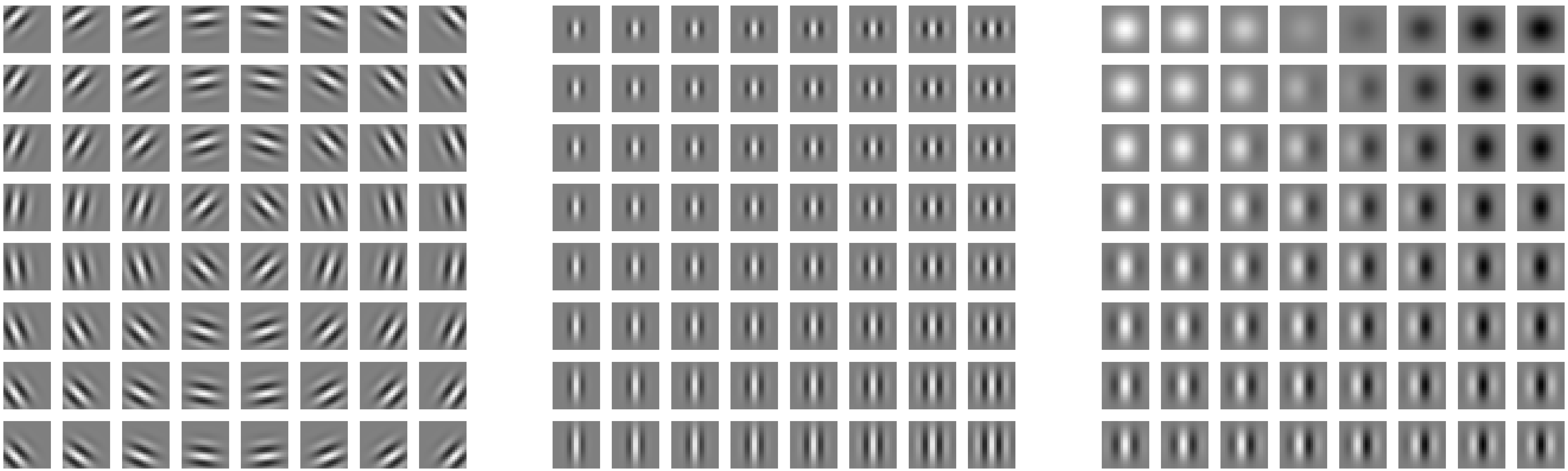


Figure 9.18

(Goodfellow 2016)

Gabor-like Learned Kernels

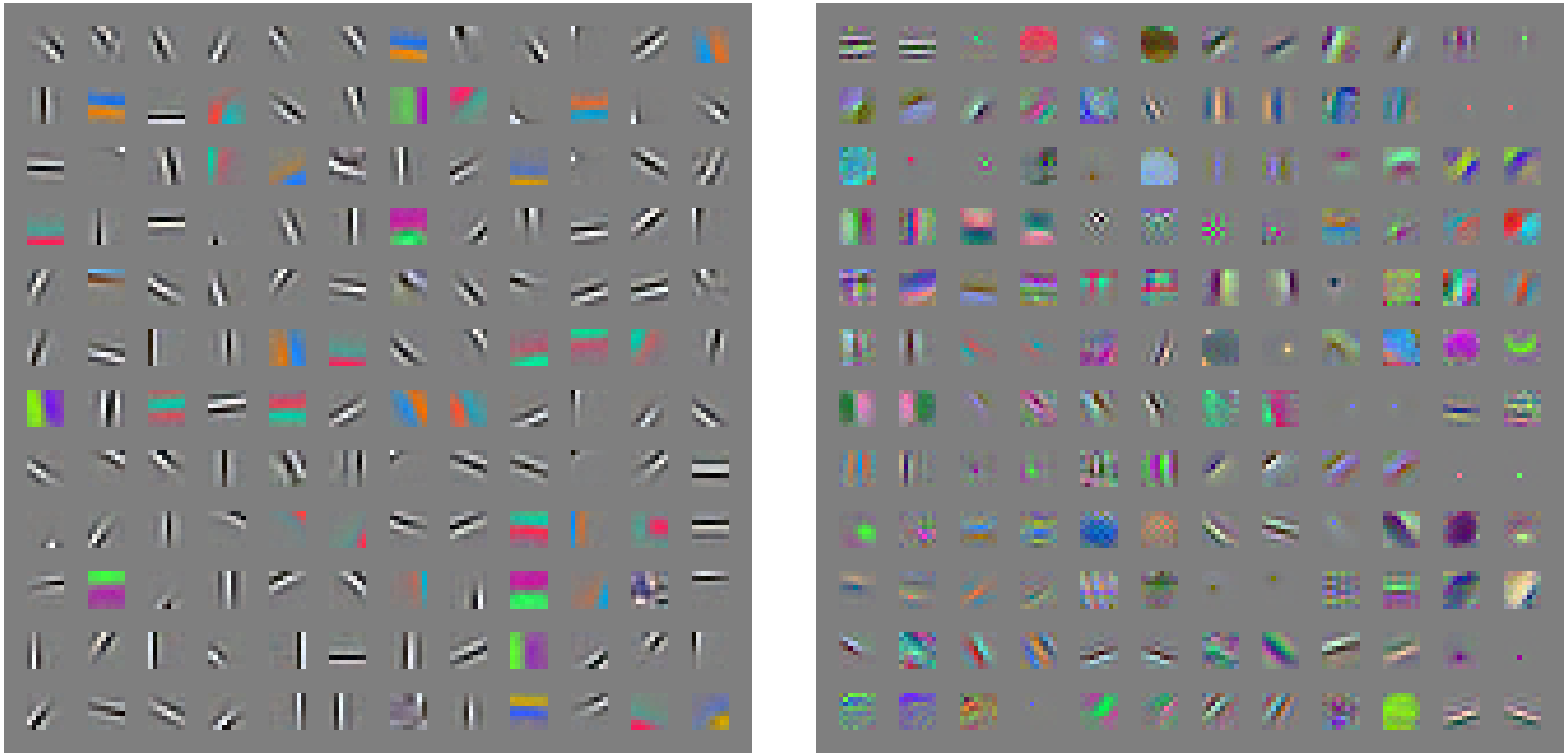


Figure 9.19

Major Architectures

- Spatial Transducer Net: input size scales with output size, all layers are convolutional
- All Convolutional Net: no pooling layers, just use strided convolution to shrink representation size
- Inception: complicated architecture designed to achieve high accuracy with low computational cost
- ResNet: blocks of layers with same spatial size, with each layer's output added to the same buffer that is repeatedly updated. Very many updates = very deep net, but without vanishing gradient.